Developing Compound Measures

Data Driven Modeling and Analysis Team
2005-2006

DDMA -- Data Driven Modeling and Analysis

We understand that good data analysis requires a synthesis of expertise from many fields

DDMA

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Expertise:

Statistics
Tomography
Functional Analysis
Geometric Measure Theory
Inverse Problems
Computational Science
Algorithmics
Numerical Methods
Optimization
Numerical Analysis
Signal Processing
High-Dimensional Data Reduction

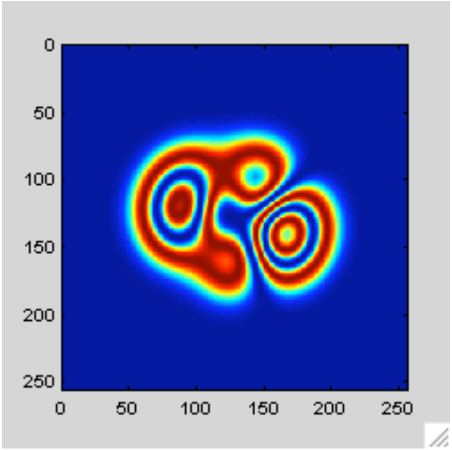
Current Applications:

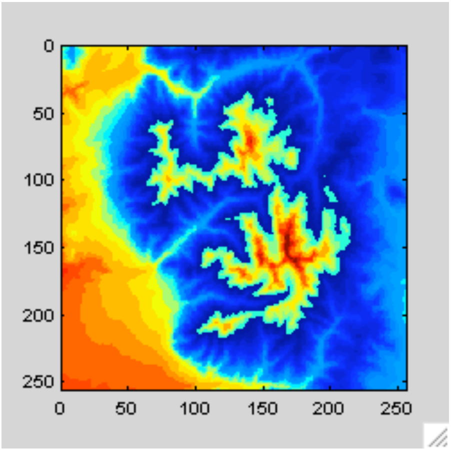
Extrapolation and Inpainting
Dimension Reduction
Object Recognition
Special X-ray Tomography
Algorithm and Prototype Software Development
Comparison Metrics
Feature Measures
Warping Transformations
Mixed-Variable Optimization

The Task

The task of comparing complex images like the ones on the right pose several challenges:

- No single measure captures the differences between the images.
- We need not only to measure the difference between the images but also to say why they are different.





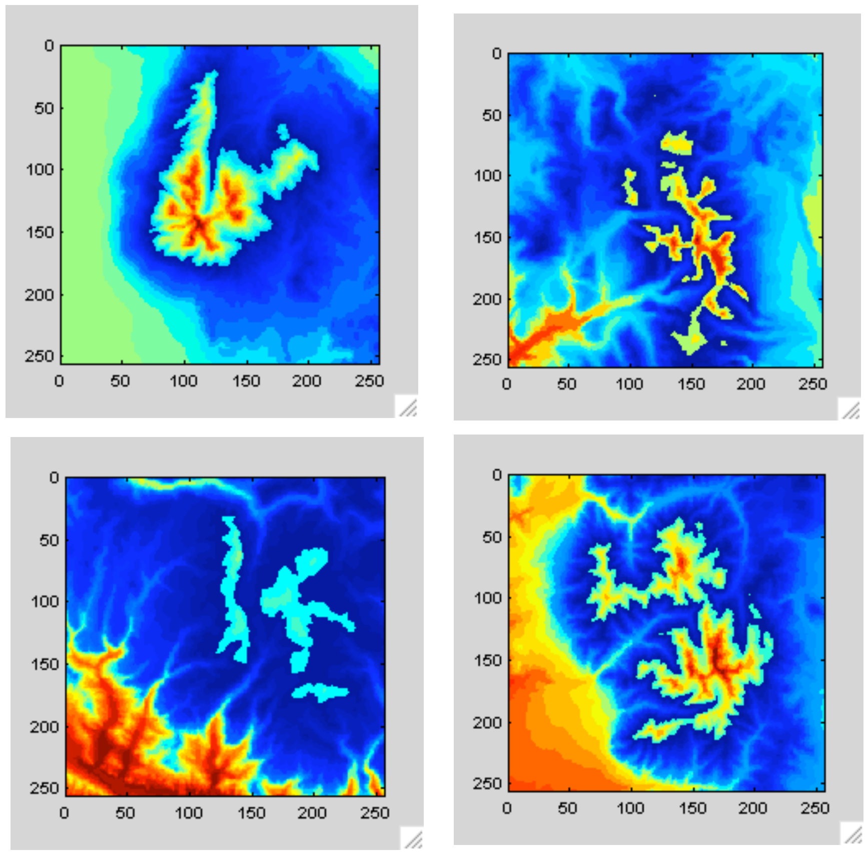
A Compound Measure

There are many gross metrics that work most of the time; however,

- We want a compound measure which
  - is composed of distinct, definable components
  - quantifies the contribution of each component
- We want a metric that codifies expert opinion in a systematic way.
- We need a process that evaluates which measures are relevant to the problem and which are not.

Example: Data Set

Complex closed curves:



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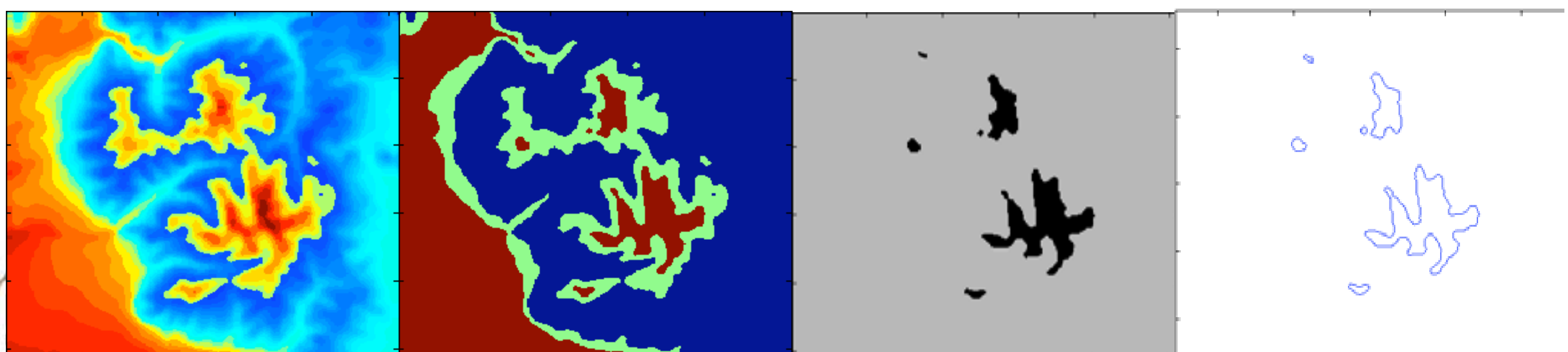
Example: IDA Pathway

- Smoothing: median filtering
- Segmentation: K-means clustering
- Deletion of boundary segments
- Boundary parametrization: trigpoly
- Characterization of curve with 13 measures: geomeasures, geomeasures2



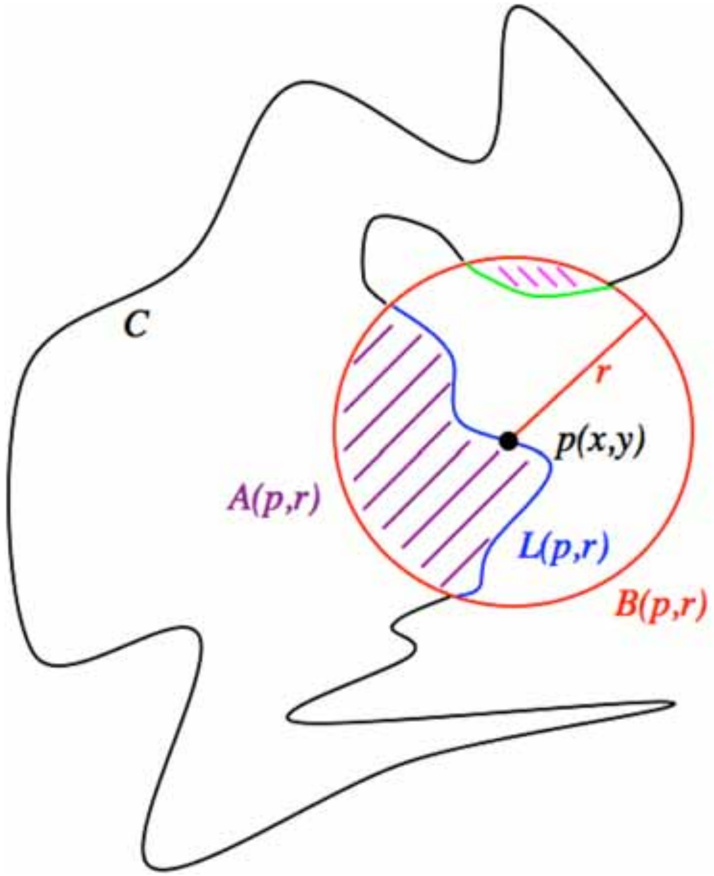
Example: A Suite of Geometric Measures (Geomeasures)

Selection of thirteen shape metrics $d_1,\ldots,d_{13}$ from IDA consisting of:

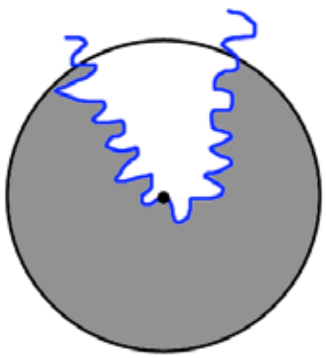
- Area
- Perimeter
- Normalized total curvature
- Number of connected components
- Variance of area of components
- Integral of 2D density ratios at 4 different scales
- Integral of isoperimetric density at 4 different scales

Method: Measures of Shape (Geomeasures)

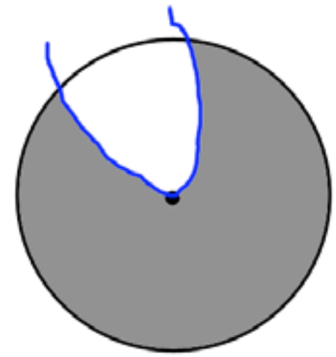
- At each point *p* on the boundary, draw a circle of radius *r*. Calculate:
  - Enclosed area *A*(*p*,*r*)
  - Enclosed length *L*(*p*,*r*)
- These values are used to generate measures of curvature:
  - Area density ratio *A*/(pi *r*^2)
  - Isoperimetric ratio *A*/*L*^2
- The measures can be taken at a range of values of *r*.



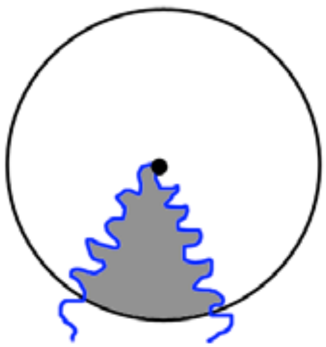
Method: Measures of Shape (Geomeasures)



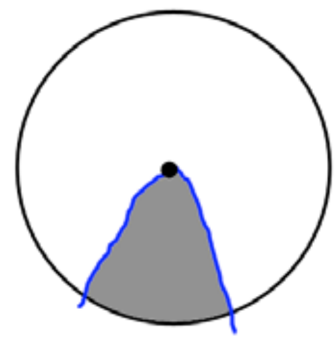
High Area Density Ratio, Low Isoperimetric Ratio



High Area Density Ratio, High Isoperimetric Ratio



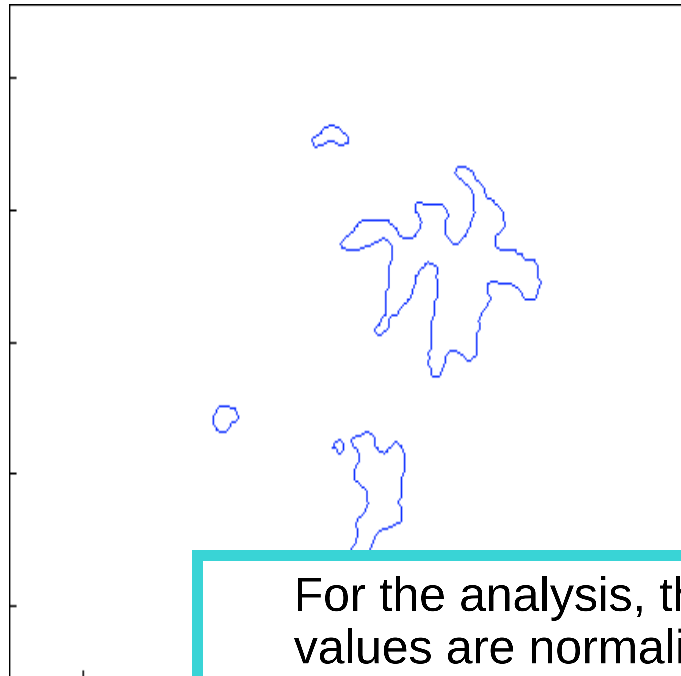
Low Area Density Ratio, Low Isoparametric Ratio



Low Area Density Ratio, High Isoperimetric Ratio

Example: Geometric Measures

Area	2772
Perimeter	625
Normalized total turns	0.2
Number of components	6
Var. of area of components	629972
Area density ratio, r=5	350
Area density ratio, r=10	299
Area density ratio, r=20	233
Area density ratio, r=40	154
Isoperimetric density, r=5	229
Isoperimetric density, r=10	131
Isoperimetric density, r=20	51
Isoperimetric density, r=40	27



For the analysis, these values are normalized to a scale of 1 to 10 based on the maximum and minimum values returned by the measure in the whole data set.

Determining the Best Combined Metric

Given:

- Expert opinion $D_1,\ldots D_k$ for the distance between k image pairs

$$(T_{1,1}, T_{1,2}), (T_{2,1}, T_{2,2}), \ldots, (T_{k,1}, T_{k,2})$$

- A collection of m shape metrics $d_1(. , .),\ldots,d_m(. , .)$

Goal:

Determine the best combined metric of the form:

$$\mathbf{d}_\alpha(\cdot,\cdot) = \sum_{i=1}^{m} \alpha_i d_i(\cdot,\cdot)$$

by choosing the constants $\alpha_1,\ldots\alpha_m$ judiciously, with the proviso that $\alpha_l >= 0$, so that the combined metric $\mathbf{d}_\alpha(. , .)$ agrees with the expert opinion as much as possible.

Getting Expert Opinion

Twenty images were selected for pair-wise comparison by experts to determine whether they were similar. Images were ranked on a scale of 0-4 as follows

0 - Indistinguishable at first glance

1 - Slightly Different

2 - Moderately Different

3 - Very different

4 - Radically different

Variational Problem

Define the matrix M as follows:

$$M = \begin{pmatrix} d_1(T_{1,1},T_{1,2}) & d_2(T_{1,1},T_{1,2}) & \ldots & d_m(T_{1,1},T_{1,2}) \\ d_1(T_{2,1},T_{2,2}) & d_2(T_{2,1},T_{2,2}) & \ldots & d_m(T_{2,1},T_{2,2}) \\ \ldots & \ldots & \ldots & \ldots \\ d_1(T_{k,1},T_{k,2}) & d_2(T_{k,1},T_{k,2}) & \ldots & d_m(T_{k,1},T_{k,2}) \end{pmatrix}$$

The least squares problem is:

$$M \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \ldots \\ \alpha_m \end{pmatrix} \approx \begin{pmatrix} D_1 \\ D_2 \\ \ldots \\ D_k \end{pmatrix}$$

Variational Problem

$$\min_{\substack{\alpha_i \geq 0 \\ i=1,\ldots,m}} \sum_{j=1}^{k} \left(D_j - \mathbf{d}_\alpha(T_{j,1}, T_{j,2})\right)^2$$

$$= \min_{\substack{\alpha_i \geq 0 \\ i=1,\ldots,m}} \underbrace{\sum_{j=1}^{k} \left(D_j - \sum_{i=1}^{m} \alpha_i d_i(T_{j,1}, T_{j,2})\right)^2}_{:=E(\alpha_1,\ldots,\alpha_m)}$$

This is a non-negative least squares problem.

Variational Problem

Variational problem: Quadratic cost, linear inequality constraint:

$$\min_{\substack{\alpha_i \geq 0 \\ i=1,\dots,m}} \|M\alpha - D\|_2^2$$

Simple Algorithm:

1. A step in the negative gradient direction:

$$\alpha \longrightarrow \alpha - (\delta t) M^T (M\alpha - D)$$

2. Projection back to feasible set:

$$\alpha_i \longrightarrow \max\{\alpha_i, 0\}$$

Note: δt is the time step size, to be chosen small enough.

Selection of metrics

- After minimization, we determine a set of alpha values for:

$$\mathbf{d}_{\alpha}(\cdot,\cdot) = \sum_{i=1}^{m} \alpha_i d_i(\cdot,\cdot)$$

- The non-zero alphas correspond to metrics that do well at making judgments that agree with the expert.

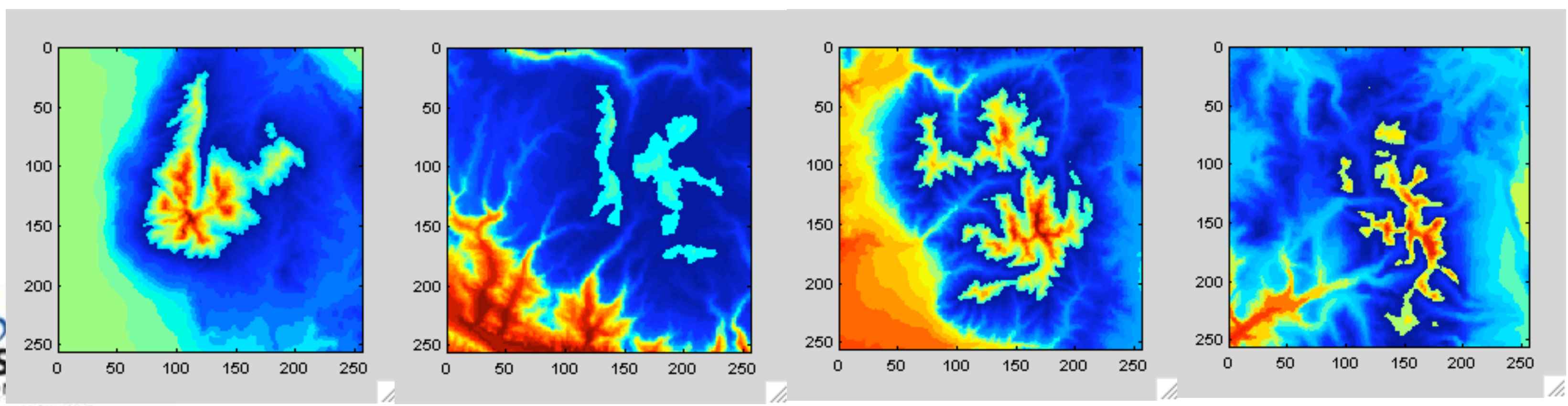
A Combined Metric: Weights on Metrics

	Alpha
Area	0.0000
Perimeter	0.0000
Normalized Total Turn (total turn/perimeter)	**0.0225**
Number of Components	**0.0522**
Variance of the Area of the Components	**0.1578**
Integral of the Area Density Ratios, r = 5	0.0000
Integral of the Area Density Ratios, r = 10	0.0000
Integral of the Area Density Ratios, r = 20	0.0000
Integral of the Area Density Ratios, r = 40	**0.0609**
Integral of the Isoperimetric Ratio, r = 5	**0.0205**
Integral of the Isoperimetric Ratio, r = 10	0.0000
Integral of the Isoperimetric Ratio, r = 20	0.0000
Integral of the Isoperimetric Ratio, r = 40	0.0000

Metrics that contribute to emulating expert opinion have a positive alpha value and are shown in blue.

Evaluating the Results: Classification

- This particular data set is derived from the contours of four peaks that were warped by three methods:
  - Poisson noise
  - Blurring
  - Sine-warping
- Expert opinion generally correlated with correct classification of the images by peak.



Evaluating the Results: Classification of the Test Data Set

Mountain #1		Mountain #2		Mountain #3		Mountain #4	
1	1	2	2	**4**	**2**	**1**	4
2	1	2	2	3	**2**	**3**	4
2	1	2	2	3	**2**	**3**	4
2	1	2	2	3	**2**	4	4
2	1	2	2	3	**2**	4	4
2	1	2	2	3	**2**	4	4
1	1	2	2	3	**2**	4	4
1	1	2	2	3	**2**	4	4
1	1	2	2	3	**2**	4	4
1	1	2	2	3	**2**	4	4
1	1	2	2	3	**2**	4	4
1	1	2	2	3	**2**	4	4
1	1	2	2	**2**	**2**	4	4
1	1	2	2	**2**	**2**	4	4
1	1	2	2	**2**	**2**	4	4
1	1	2	2	**2**	**2**	4	4
1	1	2	2	**2**	**2**	4	4
1	1	2	2	**2**	**2**	4	4

Peaks 1, 2 and 4 are reasonably well classified. Peak 3 is poorly classified.

Mis-classifications in red

Evaluating the Results: The Difference Between Expert Opinion and Combined Metric Results

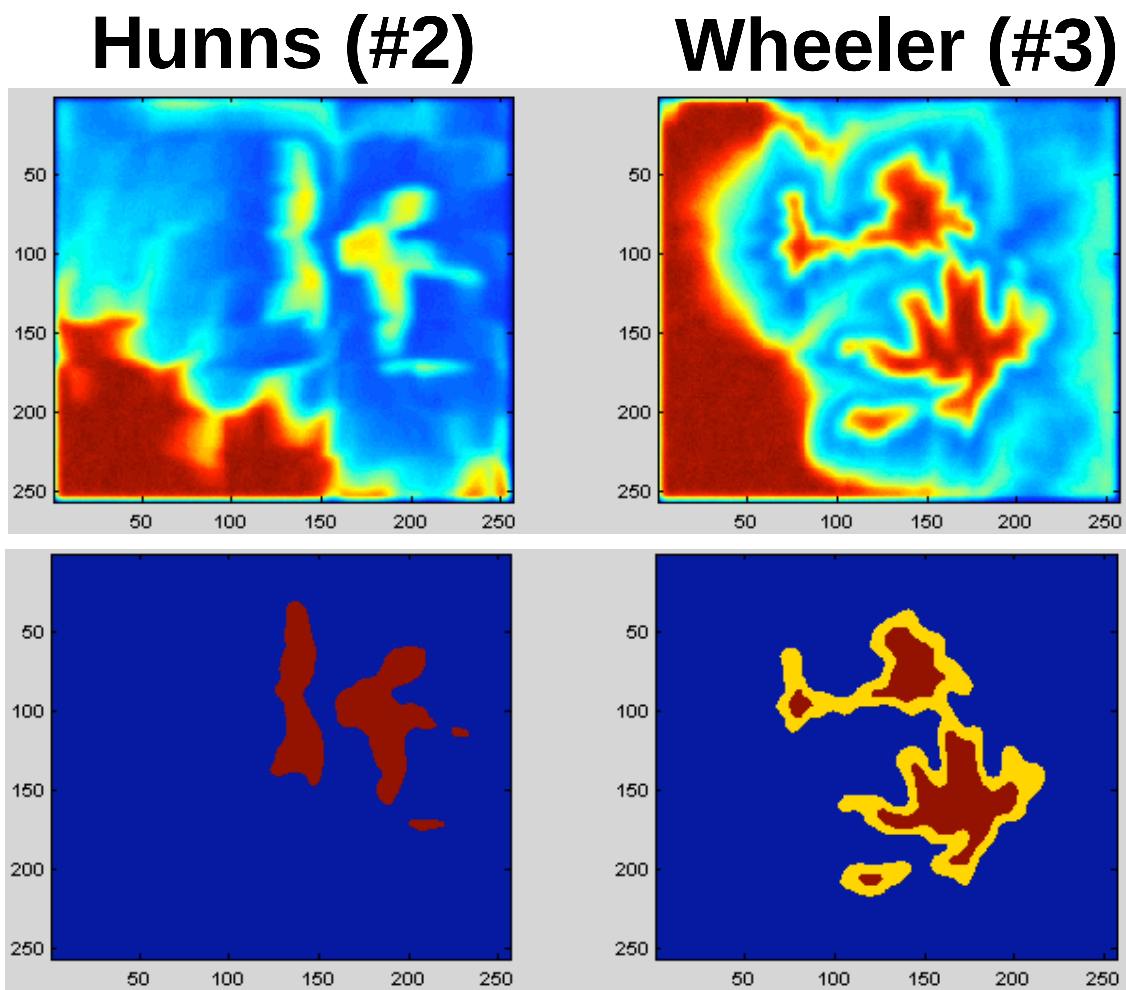
											Wheeler									
Hunns	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	1	2	1	2	1	2	1	1	0	1	4	1	2	2	1	0	1	2	2
2		0	1	1	1	1	1	1	1	1	1	4	1	2	1	1	0	1	2	2
3			0	1	1	1	2	1	1	1	1	4	1	2	2	1	0	1	2	2
4				0	1	1	1	1	1	1	1	4	1	2	2	1	0	1	2	2
5					0	1	1	0	1	1	1	4	1	2	2	1	0	1	2	2
6						0	1	1	1	0	2	2	4	3	3	3	0	1	0	0
7							0	1	1	0	2	2	4	3	3	3	1	1	1	1
8								0	1	2	2	2	4	3	3	3	1	1	1	1
9									0	1	2	2	4	3	3	4	1	1	1	1
10										0	1	1	4	3	3	3	1	1	1	1
11											0	1	1	1	1	2	2	3	2	3
12												0	2	0	2	3	2	0	0	0
13													0	1	1	1	2	2	2	2
14														0	1	1	3	2	3	3
15															0	1	2	2	3	3
16																0	1	2	2	2
17																	0	2	1	1
18																		0	1	1
19																			0	1
20																				0

Our combined metric does well on most of the peaks, but poorly emulates expert opinion with respect to the comparison between Wheeler (#3) and Hunns (#2).

Evaluating the Results: Hunns and Wheeler Peaks

Why does the measure equate Hunns and Wheeler Peak?

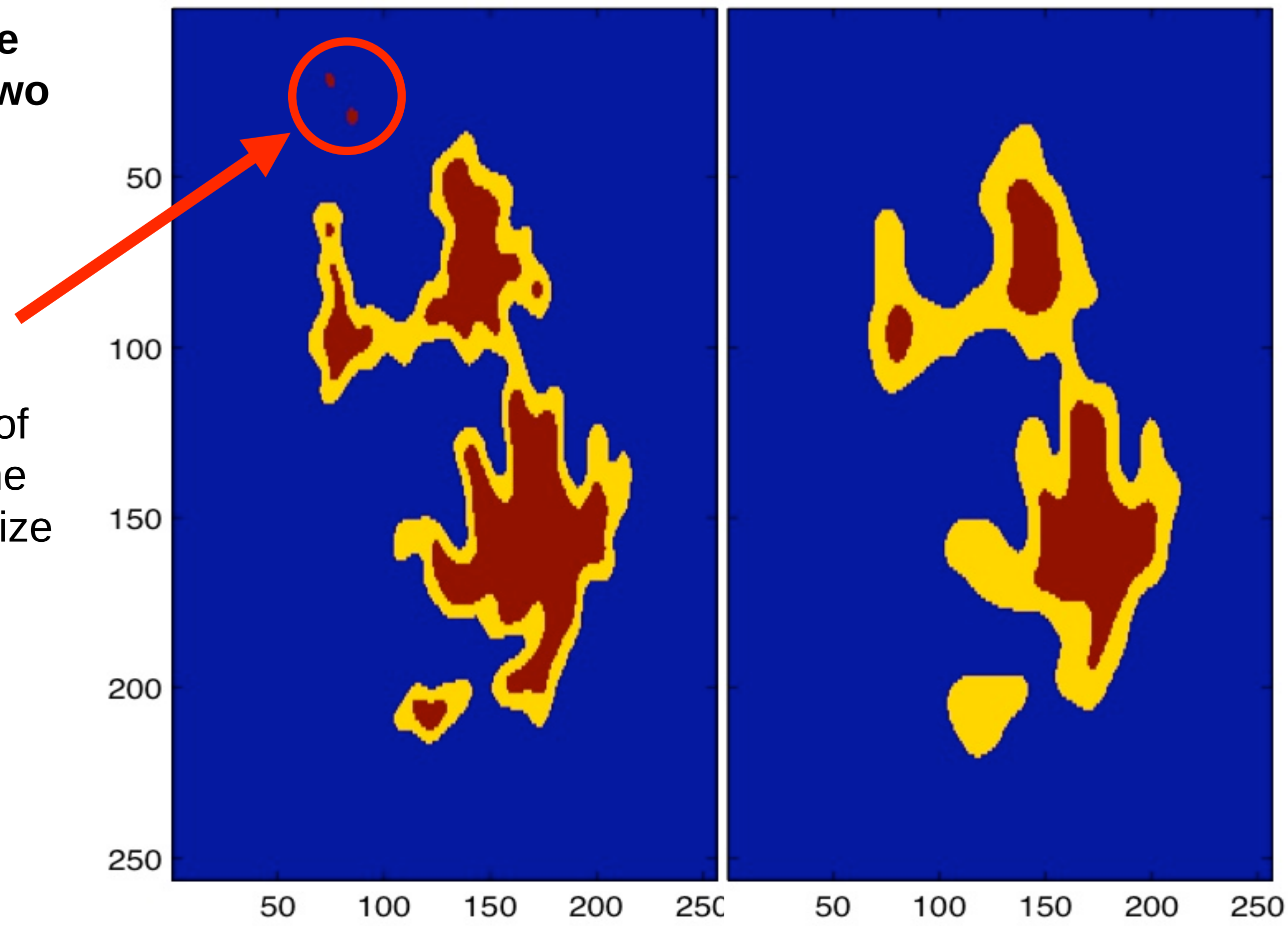
The measure correctly characterizes the number of components but fails to characterize both the shape of the components or the relative position of those components well.



Comparison of Two Perturbations of Wheeler Peak

Why does the measure distinguish between two perturbations of the same peak?

These two little islands cause the combined measure to distinguish between these images of the same mountain. The measure fails to recognize the overall similarity between the two larger sub-components and weight that similarity appropriately.



Evaluating the Results: Conclusions

- The emphasis of this analysis was not on the specific measures but on demonstrating the combined measure method.
- We used a limited number of course measures that do not adequately the characterize shape and the position of components*.
- However we have demonstrated a process that distinguishes which measures matter and produces a combined measure that:
  - is composed of distinct, definable components
  - quantifies the contribution of each component
  - codifies expert opinion in a systematic way

* Note that a measure like the L^2 measure that would have worked well in classifying this particular problem. However, the L^2 would not demonstrate what the combined measure method does well.